Factoring Trinomials Worksheet #1

1. $b^2 + 8b + 7$

   Solution:

   Signs inside the two binomials are identical and positive.

   Factors of $b^2 = b \cdot b$

   Factors of $7 = 1 \cdot 7$

   $$b^2 + 8b + 7 = (b + 1)(b + 7)$$

2. $n^2 - 11n + 10$

   Solution:

   Signs inside the two binomials are identical and negative.

   Factors of $n^2 = n \cdot n$

   Factors of $10 = 1 \cdot 10 \qquad 10 = 2 \cdot 5$

   Possible pairs of factors:

   $$(n - 1)(n - 10)$$

   $$(n - 2)(n - 5)$$

   $$n^2 - 11n + 10 = (n - 1)(n - 10)$$

3. $m^2 + m - 90$

Solution:

Signs inside the two binomials are different.

Factors of $m^2 = m \cdot m$

Factors of $90 = 1 \cdot 90 \qquad 90 = 2 \cdot 45 \qquad 90 = 3 \cdot 30$

$90 = 5 \cdot 18 \qquad 90 = 6 \cdot 15 \qquad 90 = 9 \cdot 10$

Possible pairs of factors:

$(m + 1)(m - 90)$
$(m + 90)(m - 1)$
$(m + 2)(m - 45)$
$(m + 45)(m - 2)$
$(m + 3)(m - 30)$
$(m + 30)(m - 3)$
$(m + 5)(m - 18)$
$(m + 18)(m - 5)$
$(m + 6)(m - 15)$
$(m + 15)(m - 6)$
$(m + 9)(m - 10)$
$(m + 10)(m - 9)$

Our experience should now tell us that because the sign of the cross term of the trinomial is positive, then the factor of 10 with the larger absolute value should have the plus sign. Also, the fact that the coefficient of the cross term is just 1 should tell us that the absolute values of the factors of 10 can only differ by 1. Therefore, the correct possibility if the last one. That is,

$$m^2 + m - 90 = (m + 10)(m - 9)$$

4. $n^2 + 4n - 12$

Solution:

Signs inside the two binomials are different.

Factors of $n^2 = n \cdot n$

Factors of $12 = 1 \cdot 12 \qquad 12 = 2 \cdot 6 \qquad 12 = 3 \cdot 4$

Possible pairs of factors:

$(n + 1)(n - 12)$
$(n + 12)(n - 1)$
$(n + 2)(n - 6)$
$(n + 6)(n - 2)$
$(n + 3)(n - 4)$
$(n + 4)(n - 3)$

Given the positive nature of the cross term, we know the factor of 12 with the larger absolute value in each pair in each pair must be the factor of 12 with the plus sign. Second given that the coefficient of the cross term is 4, we know the difference of the two factor of 12 must be 4. Therefore,

$$n^2 + 4n - 12 = (n + 6)(n - 2)$$

5. $q^2 - 10q + 9$

Solution:

Signs inside the two binomials are identical and negative.

Factors of $q^2 = q \cdot q$

Factors of $9 = 1 \cdot 9 \qquad 9 = 3 \cdot 3$

Possible pairs of factors:

$(q - 1)(q - 9)$
$(q - 3)(q - 3)$

Since $1 + 9 = 10$, we can deduce,

$$q^2 - 10q + 9 = (q - 1)(q - 9)$$

6. $5v^2 - 30v + 40$

Solution:

First we notice the common factor, 5, in all the terms of the trinomial. Therefore, we are now factoring,

$$5(v^2 - 6v + 8)$$

Signs inside the two binomials are identical and negative.

Factors of $v^2 = v \cdot v$

Factors of $8 = 1 \cdot 8 \qquad 8 = 2 \cdot 4$

Possible pairs of factors with common factor:

$$5(v-1)(v-8)$$
$$5(v-2)(v-4)$$

Because $2 + 4 = 6$, it should be obvious that,

$$5v^2 - 30v + 40 = 5(v-2)(v-4)$$

7. $a^2 - a - 90$

Solution:

This problem is essentially the exact same problem (variables are different but so what) as problem 3 above, except for the sign of the cross term. The solution will be the same except this time the minus sign will go with the factor of 90 with the larger absolute value. Therefore,

$$a^2 - a - 90 = (a - 10)(a + 9)$$

8. $3x^2 - 3x - 6$

Solution:

First common factor of 3 giving, $3(x^2 - x - 2)$.

Signs inside the two binomials are different.

Factors of $x^2 = x \cdot x$

Factors of $2 = 1 \cdot 2$

Possible pairs of factors with common factor:

$3(x + 1)(x - 2)$

$3(x + 2)(x - 1)$

$$3x^2 - 3x - 6 = 3(x + 1)(x - 2)$$

9. $2w^2 + 11w + 5$

Solution:

No common factor.

Signs inside the two binomials are identical and positive.

Factors of $2w^2 = w \cdot 2w$

Factors of $5 = 1 \cdot 5$

Possible pairs of factors:

$(w + 1)(2w + 5)$

$(w + 5)(2w + 1)$

Given that the cross term equals $11w$, we should see that,

$$2w^2 + 11w + 5 = (w + 5)(2w + 1)$$

10. $7y^2 + 53y + 28$

Solution:

No common factor.

Signs inside the two binomials are identical and positive.

Factors of $7y^2 = y \cdot 7y$

Factors of $28 = 1 \cdot 28 \quad 28 = 2 \cdot 14 \quad 28 = 4 \cdot 7$

Possible pairs of factors:

$$(y+1)(7y+28)$$
$$(y+28)(7y+1)$$
$$(y+2)(7y+14)$$
$$(y+14)(7y+2)$$
$$(y+4)(7y+7)$$
$$(y+7)(7y+4)$$

We want to achieve a cross term of $53y$. Therefore,

$$7y^2 + 53y + 28 = (y+7)(7y+4)$$

11. $a^2 - 4ab - x^4 + 4b^2$

Solution:

This problem is NOT one to solve by trial and error. Let's try to use our knowledge of patterns.

Rearranging the terms of the trinomial,

$$a^2 - 4ab - x^4 + 4b^2 = a^2 - 4ab + 4b^2 - x^4$$

We see a perfect square in the trinomial. Namely

$$a^2 - 4ab + 4b^2 - x^4 = (a)^2 - 2(a)(2b) + (2b)^2 - x^4$$
$$= (a-2b)^2 - x^4$$

And we recognize this last expression as the difference of two squares. Therefore,

$$(a-2b)^2 - x^4 = (a - 2b + x^2)(a - b - x^2)$$

12. $a^2 - 4b^2 + 4bc - c^2$

Solution:

Again, this problem is NOT one to solve by trial and error. Let's try to use our knowledge of patterns.

Factoring out a common term of -1, from some of the terms of the original polynomial, we get,

$$a^2 - 4b^2 + 4bc - c^2 = a^2 - (4b^2 - 4bc + c^2)$$

We see a perfect square in the trinomial. Namely

$$a^2 - (4b^2 - 4bc + c^2) = a^2 - [(2b)^2 - 2(2b)(c) + (c)^2]$$
$$= a^2 - (2b - c)^2$$

And we recognize this last expression as the difference of two squares. Therefore,

$$a^2 - (2b - c)^2 = (a + [2b - c])(a - [2b - c])$$
$$= (a + 2b - c)(a - 2b + c)$$

13. (a) Show that the difference of the squares of two consecutive integers is equal to the sum of the two integers.

Solution:

Let x = any integer

Then $x + 1 =$ the next consecutive integer

We know the pattern of the difference of any two squares. Namely,

$$a^2 - b^2 = (a+b)(a-b)$$

Suppose the two squares we were interested in were not simple a and b, but rather the specific $(x+1)$ and x. Then we would have,

$$\begin{aligned}(x+1)^2 - x^2 &= (x+1+x)(x+1-x)\\ &= (2x+1)(1)\\ &= 2x+1\\ &= x+1+x\\ (x+1)^2 - x^2 &= (x+1)+x\end{aligned}$$

And this last result is precisely the sum of the two consecutive integers.

(b) Use the result to obtain the square of 51.

Solution:

We want to find the value of 51^2. Using the result above, we can say,

$$\begin{aligned}51^2 - 50^2 &= 51 + 50 = 101\\ 51^2 - 2500 &= 101\\ 51^2 &= 101 + 2500\\ 51^2 &= 2601\end{aligned}$$